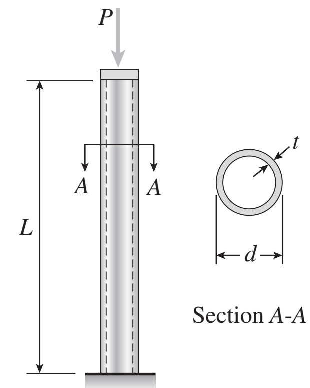
Problem 11.9-9 Determine the allowable axial load P_{allow} for a steel pipe column that is fixed at the base and free at the top (see figure) for each of the following lengths: $L = 6$ ft, 9 ft, 12 ft, and 15 ft. The column has outside diameter $d = 6.625$ in. and wall thickness $t = 0.280$ in. (Assume $E = 29{,}000$ ksi and $\sigma_Y = 36$ ksi.)



Probs. 11.9-9 through 11.9-12

Solution 11.9-9 Steel pipe column

Fixed-free column ($K = 2$).

Use AISC formulas.

$d_2 = 6.625$ in. $\quad t = 0.280$ in. $\quad d_1 = 6.065$ in.

$$A = \frac{\pi}{4}(d_2^2 - d_1^2) = 5.5814\text{ in.}^2$$

$$I = \frac{\pi}{64}(d_2^4 - d_1^4) = 28.142\text{ in.}^4$$

$$r = \sqrt{\frac{I}{A}} = 2.2455 \qquad \left(\frac{KL}{r}\right)_{max} = 200$$

$E = 29{,}000$ ksi $\quad \sigma_Y = 36$ ksi

Eq.(11-76): $$\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = 126.1$$

$$L_c = 126.1\frac{r}{k} = 141.6\text{ in.} = 11.8\text{ ft}$$

L	6 ft	9 ft	12 ft	15 ft
KL/r	64.13	96.19	128.3	160.3
n_1 (Eq. 11-79)	1.841	1.897	–	–
n_2 (Eq. 11-80)	–	–	1.917	1.917
σ_{allow}/σ_Y (Eq. 11-81)	0.4730	0.3737	–	–
σ_{allow}/σ_Y (Eq. 11-82)	–	–	0.2519	0.1614
σ_{allow} (ksi)	17.03	13.45	9.078	5.810
$P_{allow} = A\,\sigma_{allow}$	95.0 k	75.1 k	50.7 k	32.4 k

Problem 11.9-10 Determine the allowable axial load P_{allow} for a steel pipe column that is fixed at the base and free at the top (see figure) for each of the following lengths: $L = 2.6$ m, 2.8 m, 3.0 m, and 3.2 m. The column has outside diameter $d = 140$ mm and wall thickness $t = 7$ mm. (Assume $E = 200$ GPa and $\sigma_Y = 250$ MPa.)

Solution 11.9-10 Steel pipe column

Fixed-free column ($K = 2$).

Use AISC formulas.

$d_2 = 140$ mm $\quad t = 7.0$ mm $\quad d_1 = 126$ mm

$$A = \frac{\pi}{4}(d_2^2 - d_1^2) = 2924.8\text{ mm}^2$$

$$I = \frac{\pi}{64}(d_2^4 - d_1^4) = 6.4851 \times 10^6\text{ mm}^4$$

$$r = \sqrt{\frac{I}{A}} = 47.09\text{ mm} \qquad \left(\frac{KL}{r}\right)_{max} = 200$$

$E = 200$ GPa $\quad \sigma_Y = 250$ MPa

Eq.(11-76): $$\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = 125.7$$

$$L_c = 125.7\frac{r}{K} = 2959\text{ mm} = 2.959\text{ m}$$

L	2.6 m	2.8 m	3.0 m	3.2 m
KL/r	110.4	118.9	127.4	135.9
n_1 (Eq. 11-79)	1.911	1.916	–	–
n_2 (Eq. 11-80)	–	–	1.917	1.917
σ_{allow}/σ_Y (Eq. 11-81)	0.3212	0.2882	–	–
σ_{allow}/σ_Y (Eq. 11-82)	–	–	0.2537	0.2230
σ_{allow} (MPa)	80.29	72.06	63.43	55.75
$P_{allow} = A\,\sigma_{allow}$	235 kN	211 kN	186 kN	163 kN

Problem 11.9-11 Determine the maximum permissible length L_{max} for a steel pipe column that is fixed at the base and free at the top and must support an axial load $P = 40$ k (see figure). The column has outside diameter $d = 4.0$ in., wall thickness $t = 0.226$ in., $E = 29{,}000$ ksi, and $\sigma_Y = 42$ ksi.

Solution 11.9-11 Steel pipe column

Fixed-free column ($K = 2$). $P = 40$ k

Use AISC formulas.

$d_2 = 4.0$ in. $t = 0.226$ in. $d_1 = 3.548$ in.

$$A = \frac{\pi}{4}(d_2^2 - d_1^2) = 2.6795 \text{ in.}$$

$$I = \frac{\pi}{64}(d_2^4 - d_1^4) = 4.7877 \text{ in.}^4$$

$$r = \sqrt{\frac{I}{A}} = 1.3367 \qquad \left(\frac{KL}{r}\right)_{max} = 200$$

$E = 29{,}000$ ksi $\sigma_Y = 42$ ksi

$$\text{Eq.(11-76):} \quad \left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = 116.7$$

$$L_c = 116.7\,\frac{r}{K} = 78.03 \text{ in.} = 6.502 \text{ ft}$$

Select trial values of the length L and calculate the corresponding values of P_{allow} (see table). Interpolate between the trial values to obtain the value of L that produces $P_{allow} = P$.

Note: If $L < L_c$, use Eqs. (11-79) and (11-81).
If $L > L_c$, use Eqs. (11-80) and (11-82).

L(ft)	5.20	5.25	5.23
KL/r	93.86	94.26	93.90
n_1 (Eq. 11-79)	1.903	1.904	1.903
n_2 (Eq. 11-80)	–	–	–
σ_{allow}/σ_Y (Eq. 11-81)	0.3575	0.3541	0.3555
σ_{allow}/σ_Y (Eq. 11-82)	–	–	–
σ_{allow} (ksi)	15.02	14.87	14.93
$P_{allow} = A\,\sigma_{allow}$	40.2 k	39.8 k	40.0 k

For $P = 40$ k, $L_{max} = 5.23$ ft ⟵

Problem 11.9-12 Determine the maximum permissible length L_{max} for a steel pipe column that is fixed at the base and free at the top and must support an axial load $P = 500$ kN (see figure). The column has outside diameter $d = 200$ mm, wall thickness $t = 10$ mm, $E = 200$ GPa, and $\sigma_Y = 250$ MPa.

Solution 11.9-12 Steel pipe column

Fixed-free column ($K = 2$). $P = 500$ kN

Use AISC formulas.

$d_2 = 200$ mm $t = 10$ mm $d_1 = 180$ mm

$$A = \frac{\pi}{4}(d_2^2 - d_1^2) = 5{,}969.0 \text{ mm}^2$$

$$I = \frac{\pi}{64}(d_2^4 - d_1^4) = 27.010 \times 10^6 \text{ mm}^4$$

$$r = \sqrt{\frac{I}{A}} = 67.27 \text{ mm} \quad \left(\frac{KL}{r}\right)_{max} = 200$$

$E = 200$ GPa $\sigma_Y = 250$ MPa

$$\text{Eq. (11-76):} \left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = 125.7$$

$$L_c = 125.7\,\frac{r}{K} = 4.226 \text{ m}$$

Select trial values of the length L and calculate the corresponding values of P_{allow} (see table). Interpolate between the trial values to obtain the value of L that produces $P_{allow} = P$.

Note: If $L < L_c$, use Eqs. (11-79) and (11-81).
If $L > L_c$, use Eqs. (11-80) and (11-82).

L(m)	3.55	3.60	3.59
KL/r	105.5	107.0	106.7
n_1 (Eq. 11-79)	1.908	1.909	1.909
n_2 (Eq. 11-80)	–	–	–
σ_{allow}/σ_Y (Eq. 11-81)	0.3393	0.3338	0.3349
σ_{allow}/σ_Y (Eq. 11-82)	–	–	–
σ_{allow} (MPa)	84.83	83.46	83.74
$P_{allow} = A\,\sigma_{allow}$	506 kN	498 kN	500 kN

For $P = 500$ kN, $L = 3.59$ m ⟵

Problem 11.9-13 A steel pipe column with *pinned ends* supports an axial load $P = 21$ k. The pipe has outside and inside diameters of 3.5 in. and 2.9 in., respectively. What is the maximum permissible length L_{max} of the column if $E = 29{,}000$ ksi and $\sigma_Y = 36$ ksi?

Solution 11.9-13 **Steel pipe column**

Pinned ends ($K = 1$). $P = 21$ k

Use AISC formulas.

$d_2 = 3.5$ in. $t = 0.3$ in. $d_1 = 2.9$ in.

$$A = \frac{\pi}{4}(d_2^2 - d_1^2) = 3.0159 \text{ in.}^2$$

$$I = \frac{\pi}{64}(d_2^4 - d_1^4) = 3.8943 \text{ in.}^4$$

$$r = \sqrt{\frac{I}{A}} = 1.1363 \text{ in.} \qquad \left(\frac{L}{r}\right)_{max} = 200$$

$E = 29{,}000$ ksi $\sigma_Y = 36$ ksi

$$\text{Eq. (11-76): } \left(\frac{L}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = 126.1$$

$L_c = 126.1\, r = 143.3$ in. $= 11.9$ ft

Select trial values of the length L and calculate the corresponding values of P_{allow} (see table). Interpolate between the trial values to obtain the value of L that produces $P_{allow} = P$.

Note: If $L < L_c$, use Eqs. (11-79) and (11-81).

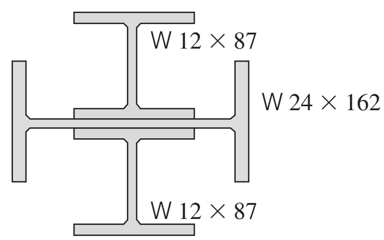
If $L > L_c$, use Eqs. (11-80) and (11-82).

L(ft)	13.8	13.9	14.0
L/r	145.7	146.8	147.8
n_1 (Eq. 11-79)	–	–	–
n_2 (Eq. 11-80)	1.917	1.917	1.917
σ_{allow}/σ_Y (Eq. 11-81)	–	–	–
σ_{allow}/σ_Y (Eq. 11-82)	0.1953	0.1925	0.1898
σ_{allow} (ksi)	7.031	6.931	6.832
$P_{allow} = A\,\sigma_{allow}$	21.2 k	20.9 k	20.6 k

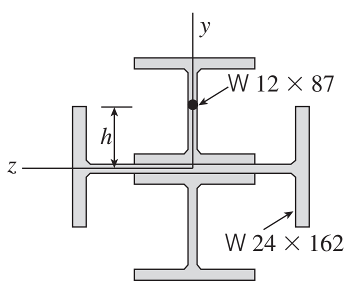
For $P = 21$ k, $L = 13.9$ ft ←

Problem 11.9-14 The steel columns used in a college recreation center are 55 ft long and are formed by welding three wide-flange sections (see figure). The columns are pin-supported at the ends and may buckle in any direction.

Calculate the allowable load P_{allow} for one column, assuming $E = 29{,}000$ ksi and $\sigma_Y = 36$ ksi.



Solution 11.9-14 **Pinned-end column ($K = 1$)**



$L = 55$ ft $= 660$ in.

$E = 29{,}000$ ksi

$\sigma_Y = 36$ ksi

W 12 × 87

$A = 25.6$ in.^{2} $d = 12.53$ in.

$I_1 = 740$ in.^{4} $I_2 = 241$ in.^{4}

W 24 × 162

$A = 47.7$ in.^{2} $t_w = 0.705$ in.

$I_1 = 5170$ in.^{4} $I_2 = 443$ in.^{4}

For the entire cross section

$A = 2\,(25.6) + 47.7 = 98.9$ in.^{2}

$I_Y = 2\,(241) + 5170 = 5652$ in.^{4}

$h = d/2 + t_w/2 = 6.6175$ in.

$I_z = 443 + 2\,[740 + (25.6)(6.6175)^2] = 4165$ in.^{4}

$$\text{min. } r = \sqrt{\frac{I_z}{A}} = \sqrt{\frac{4165}{98.9}} = 6.489 \text{ in.}$$

$$\text{Eq. (11-76): } \left(\frac{L}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = 126.1$$

$$\frac{L}{r} = \frac{660 \text{ in.}}{6.489 \text{ in.}} = 101.7 \qquad \frac{L}{r} < \left(\frac{L}{r}\right)_c$$

∴ Use Eqs. (11-79) and (11-81).

From Eq. (11-79): $n_1 = 1.904$

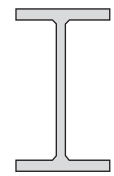
From Eq. (11-81): $\sigma_{allow}/\sigma_Y = 0.3544$

$\sigma_{allow} = 0.3544\,\sigma_Y = 12.76$ ksi

$P_{allow} = \sigma_{allow} A = (12.76 \text{ ksi})(98.9 \text{ in.}^2)$

$= 1260$ k ←

Problem 11.9-15 A W 8 × 28 steel wide-flange column with pinned ends carries an axial load P. What is the maximum permissible length L_{max} of the column if (a) $P = 50$ k, and (b) $P = 100$ k? (Assume $E = 29{,}000$ ksi and $\sigma_Y = 36$ ksi.)



Probs. 11.9-15 and 11.9-16

Solution 11.9-15 Steel wide-flange column

Pinned ends ($K = 1$).

Buckling about axis 2-2 (see Table E-1).

Use AISC formulas.

W 8 × 28 $\quad A = 8.25 \text{ in.}^2 \quad r_2 = 1.62$ in.

$E = 29{,}000$ ksi $\quad \sigma_Y = 36$ ksi $\quad \left(\frac{L}{r}\right)_{max} = 200$

Eq. (11-76): $\left(\frac{L}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = 126.1$

$L_c = 126.1\,r = 204.3$ in. $= 17.0$ ft

For each load P, select trial values of the length L and calculate the corresponding values of P_{allow} (see table). Interpolate between the trial values to obtain the value of L that produces $P_{allow} = P$.

Note: If $L < L_c$, use Eqs. (11-79) and (11-81).

If $L > L_c$, use Eqs. (11-80) and (11-82).

L (ft)	21.0	21.5	21.2
L/r	155.6	159.3	157.0
n_1 (Eq. 11-79)	–	–	–
n_2 (Eq. 11-80)	1.917	1.917	1.917
σ_{allow}/σ_Y (Eq. 11-81)	–	–	–
σ_{allow}/σ_Y (Eq. 11-82)	0.1714	0.1635	0.1682
σ_{allow} (ksi)	6.171	5.888	6.056
$P_{allow} = A\,\sigma_{allow}$	50.9 k	48.6 k	50.0 k

(a) $P = 50$ k

For $P = 50$ k, $\quad L_{max} = 21.2$ ft ←

(b) $P = 100$ k

L (ft)	14.3	14.4	14.5	
L/r	105.9	106.7	107.4	
n_1 (Eq. 11-79)	1.908	1.908	1.909	
n_2 (Eq. 11-80)	–	–	–	
σ_{allow}/σ_Y (Eq. 11-81)	0.3393	0.3366	0.3338	
σ_{allow}/σ_Y (Eq. 11-82)	–	–	–	
σ_{allow} (ksi)	12.21	12.12	12.02	
$P_{allow} = A\,\sigma_{allow}$	100.8 k	100.0 k	99.2 k	k

For $P = 100$ k, $\quad L_{max} = 14.4$ ft ←

Problem 11.9-16 A W 10 × 45 steel wide-flange column with pinned ends carries an axial load P. What is the maximum permissible length L_{max} of the column if (a) $P = 125$ k, and (b) $P = 200$ k? (Assume $E = 29{,}000$ ksi and $\sigma_Y = 42$ ksi.)

Solution 11.9-16 Steel wide-flange column

Pinned ends ($K = 1$).

Buckling about axis 2-2 (see Table E-1).

Use AISC formulas.

W 10 × 45 $A = 13.3 \text{ in.}^2$ $r_2 = 2.01$ in.

$$E = 29{,}000 \text{ ksi} \quad \sigma_Y = 42 \text{ ksi} \quad \left(\frac{L}{r}\right)_{max} = 200$$

$$\text{Eq. (11-76): } \left(\frac{L}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = 116.7$$

$L_c = 116.7\, r = 235 \text{ in.} = 19.6 \text{ ft}$

For each load P, select trial values of the length L and calculate the corresponding values of P_{allow} (see table). Interpolate between the trial values to obtain the value of L that produces $P_{allow} = P$.

Note: If $L < L_c$, use Eqs. (11-79) and (11-81).

If $L > L_c$, use Eqs. (11-80) and (11-82).

(a) $P = 125$ k

L (ft)	21.0	21.1	21.2
L/r	125.4	126.0	126.6
n_1 (Eq. 11-79)	–	–	–
n_2 (Eq. 11-80)	1.917	1.917	1.917
σ_{allow}/σ_Y (Eq. 11-81)	–	–	–
σ_{allow}/σ_Y (Eq. 11-82)	0.2202	0.2241	0.2220
σ_{allow} (ksi)	9.500	9.411	9.322
$P_{allow} = A\,\sigma_{allow}$	126.4 k	125.2 k	124.0 k

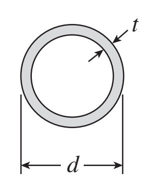
For $P = 125$ k, $L_{max} = 21.1$ ft ⟵

(b) $P = 200$ k

L(ft)	15.5	15.6	15.7
L/r	92.54	93.13	93.73
n_1 (Eq. 11-79)	1.902	1.902	1.903
n_2 (Eq. 11-80)	–	–	–
σ_{allow}/σ_Y (Eq. 11-81)	0.3607	0.3584	0.3561
σ_{allow}/σ_Y (Eq. 11-82)	–	–	–
σ_{allow} (ksi)	15.15	15.05	14.96
$P_{allow} = A\,\sigma_{allow}$	201.5 k	200.2 k	198.9 k

For $P = 200$ k, $L_{max} = 15.6$ ft ⟵

Problem 11.9-17 Find the required outside diameter d for a steel pipe column (see figure) of length $L = 20$ ft that is pinned at both ends and must support an axial load $P = 25$ k. Assume that the wall thickness t is equal to $d/20$. (Use $E = 29{,}000$ ksi and $\sigma_Y = 36$ ksi.)



Probs. 11.9-17 through 11.9-20

Solution 11.9-17 Pipe column

Pinned ends ($K = 1$).

$L = 20 \text{ ft} = 240 \text{ in.}$ $P = 25$ k

d = outside diameter $t = d/20$

$E = 29{,}000$ ksi $\sigma_Y = 36$ ksi

$$A = \frac{\pi}{4}[d^2 - (d - 2t)^2] = 0.14923\, d^2$$

$$I = \frac{\pi}{64}[d^4 - (d - 2t)^4] = 0.016881\, d^4$$

$$r = \sqrt{\frac{I}{A}} = 0.33634\, d$$

$$\left(\frac{L}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = 126.1 \qquad L_c = (126.1)r$$

Select various values of diameter d until we obtain $P_{allow} = P$.

If $L \leq L_c$, Use Eqs. (11-79) and (11-81).

If $L \geq L_c$, Use Eqs. (11-80) and (11-82).

For $P = 25$ k, $d = 4.89$ in. ←

d (in.)	4.80	4.90	5.00
A (in.^{2})	3.438	3.583	3.731
I (in.^{4})	8.961	9.732	10.551
r (in.)	1.614	1.648	1.682
L_c (in.)	204	208	212
L/r	148.7	145.6	142.7
n_2 (Eq. 11-80)	23/12	23/12	23/12
σ_{allow}/σ_Y (Eq. 11-82)	0.1876	0.1957	0.2037
σ_{allow} (ksi)	6.754	7.044	7.333
$P_{allow} = A\,\sigma_{allow}$	23.2 k	25.2 k	27.4 k

Problem 11.9-18 Find the required outside diameter d for a steel pipe column (see figure) of length $L = 3.5$ m that is pinned at both ends and must support an axial load $P = 130$ kN. Assume that the wall thickness t is equal to $d/20$. (Use $E = 200$ GPa and $\sigma_Y = 275$ MPa).

Solution 11.9-18 Pipe column

Pinned ends ($K = 1$).

$L = 3.5$ m $\quad P = 130$ kN

$d =$ outside diameter $\quad t = d/20$

$E = 200$ GPa $\quad \sigma_Y = 275$ MPa

$$A = \frac{\pi}{4}[d^2 - (d - 2t)^2] = 0.14923\,d^2$$

$$I = \frac{\pi}{64}[d^4 - (d - 2t)^4] = 0.016881\,d^4$$

$$r = \sqrt{\frac{I}{A}} = 0.33634\,d$$

$$\left(\frac{L}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = 119.8 \qquad L_c = (119.8)r$$

Select various values of diameter d until we obtain $P_{allow} = P$.

If $L \leq L_c$, Use Eqs. (11-79) and (11-81).

If $L \geq L_c$, Use Eqs. (11-80) and (11-82).

d (mm)	98	99	100
A (mm^2)	1433	1463	1492
I (mm^4)	1557×10^3	1622×10^3	1688×10^3
r (mm)	32.96	33.30	33.64
L_c (mm)	3950	3989	4030
L/r	106.2	105.1	104.0
n_1 (Eq. 11-79)	1.912	1.911	1.910
σ_{allow}/σ_Y (Eq. 11-81)	0.3175	0.3219	0.3263
σ_{allow} (MPa)	87.32	88.53	89.73
$P_{allow} = A\,\sigma_{allow}$	125.1 kN	129.5 kN	133.9 kN

For $P = 130$ kN, $d = 99$ mm ←

Problem 11.9-19 Find the required outside diameter d for a steel pipe column (see figure) of length $L = 11.5$ ft that is pinned at both ends and must support an axial load $P = 80$ k. Assume that the wall thickness t is 0.30 in. (Use $E = 29{,}000$ ksi and $\sigma_Y = 42$ ksi.)

Solution 11.9-19 Pipe column

Pinned ends ($K = 1$).

$L = 11.5$ ft $= 138$ in. $\quad P = 80$ k

$d =$ outside diameter $\quad t = 0.30$ in.

$E = 29{,}000$ ksi $\quad \sigma_Y = 42$ ksi

$$A = \frac{\pi}{4}[d^2 - (d - 2t)^2]$$

$$I = \frac{\pi}{64}[d^4 - (d - 2t)^4] \qquad r = \sqrt{\frac{I}{A}}$$

$$\left(\frac{L}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = 116.7 \qquad L_c = (116.7)r$$

Select various values of diameter d until we obtain $P_{allow} = P$.

If $L \le L_c$, Use Eqs. (11-79) and (11-81).
If $L \ge L_c$, Use Eqs. (11-80) and (11-82).

For $P = 80$ k, $d = 5.23$ in. ⟵

d (in.)	5.20	5.25	5.30
A (in.^{2})	4.618	4.665	4.712
I (in.^{4})	13.91	14.34	14.78
r (in.)	1.736	1.753	1.771
L_c (in.)	203	205	207
L/r	79.49	78.72	77.92
n_1 (Eq. 11-79)	1.883	1.881	1.880
σ_{allow}/σ_Y (Eq. 11-81)	0.4079	0.4107	0.4133
σ_{allow} (ksi)	17.13	17.25	17.36
$P_{allow} = A\,\sigma_{allow}$	79.1 k	80.5 k	81.8 k

Problem 11.9-20 Find the required outside diameter d for a steel pipe column (see figure) of length $L = 3.0$ m that is pinned at both ends and must support an axial load $P = 800$ kN. Assume that the wall thickness t is 9 mm. (Use $E = 200$ GPa and $\sigma_Y = 300$ MPa.)

Solution 11.9-20 Pipe column

Pinned ends ($K = 1$).
$L = 3.0$ m $\quad P = 800$ kN
d = outside diameter $\quad t = 9.0$ mm
$E = 200$ GPa $\quad \sigma_Y = 300$ MPa

$$A = \frac{\pi}{4}[d^2 - (d-2t)^2]$$

$$I = \frac{\pi}{64}[d^4 - (d-2t)^4] \qquad r = \sqrt{\frac{I}{A}}$$

$$\left(\frac{L}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = 114.7 \qquad L_c = (114.7)r$$

Select various values of diameter d until we obtain $P_{allow} = P$.

If $L \le L_c$, Use Eqs. (11-79) and (11-81).
If $L \ge L_c$, Use Eqs. (11-80) and (11-82).

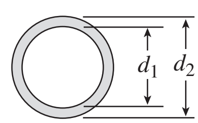
d (mm)	193	194	195
A (mm^2)	5202	5231	5259
I (mm^4)	20.08×10^6	22.43×10^6	22.80×10^6
r (mm)	65.13	65.48	65.84
L_c (mm)	7470	7510	7550
L/r	46.06	45.82	45.57
n_1 (Eq. 11-79)	1.809	1.809	1.808
σ_{allow}/σ_Y (Eq. 11-81)	0.5082	0.5087	0.5094
σ_{allow} (MPa)	152.5	152.6	152.8
$P_{allow} = A\,\sigma_{allow}$	793.1 kN	798.3 kN	803.8 kN

For $P = 800$ kn, $d = 194$ mm ⟵

Aluminum Columns

Problem 11.9-21 An aluminum pipe column (alloy 2014-T6) with pinned ends has outside diameter $d_2 = 5.60$ in. and inside diameter $d_1 = 4.80$ in. (see figure).

Determine the allowable axial load P_{allow} for each of the following lengths: $L = 6$ ft, 8 ft, 10 ft, and 12 ft.



Probs. 11.9-21 through 11.9-24

Solution 11.9-21 Aluminum pipe column

Alloy 2014-T6
Pinned ends ($K = 1$).
$d_2 = 5.60$ in.
$d_1 = 4.80$ in.

$$A = \frac{\pi}{4}(d_2^2 - d_1^2) = 6.535 \text{ in.}^2$$

$$I = \frac{\pi}{64}(d_2^2 - d_1^2) = 22.22 \text{ in.}^4$$

$$r = \sqrt{\frac{I}{A}} = 1.844 \text{ in.}$$

Use Eqs. (11-84 *a* and *b*):

$\sigma_{allow} = 30.7 - 0.23\,(L/r)$ ksi $\quad L/r \leq 55$

$\sigma_{allow} = 54{,}000/(L/r)^2$ ksi $\quad L/r \geq 55$

L (ft)	6 ft	8 ft	10 ft	12 ft
L/r	39.05	52.06	65.08	78.09
σ_{allow} (ksi)	21.72	18.73	12.75	8.86
$P_{allow} = \sigma_{allow} A$	142 k	122 k	83 k	58 k

Problem 11.9-22 An aluminum pipe column (alloy 2014-T6) with pinned ends has outside diameter $d_2 = 120$ mm and inside diameter $d_1 = 110$ mm (see figure).

Determine the allowable axial load P_{allow} for each of the following lengths: $L = 1.0$ m, 2.0 m, 3.0 m, and 4.0 m.

(*Hint:* Convert the given data to USCS units, determine the required quantities, and then convert back to SI units.)

Solution 11.9-22 Aluminum pipe column

Alloy 2014-T6

Pinned ends ($K = 1$).

$d_2 = 120 \text{ mm} = 4.7244 \text{ in.}$

$d_1 = 110 \text{ mm} = 4.3307 \text{ in.}$

$$A = \frac{\pi}{4}(d_2^2 - d_1^2) = 2.800 \text{ in.}^2$$

$$I = \frac{\pi}{64}(d_2^2 - d_1^2) = 7.188 \text{ in.}^4$$

$$r = \sqrt{\frac{I}{A}} = 40.697 \text{ mm} = 1.6022 \text{ in.}$$

Use Eqs. (11-84 *a* and *b*):

$\sigma_{allow} = 30.7 - 0.23\,(L/r)$ ksi $\quad L/r \leq 55$

$\sigma_{allow} = 54{,}000/(L/r)^2$ ksi $\quad L/r \geq 55$

L (m)	1.0 m	2.0 m	3.0 m	4.0 m
L (in.)	39.37	78.74	118.1	157.5
L/r	24.58	49.15	73.73	98.30
σ_{allow} (ksi)	25.05	19.40	9.934	5.588
$P_{allow} = \sigma_{allow} A$	70.14 k	54.31 k	27.81 k	15.65 k
P_{allow} (kN)	312 kN	242 kN	124 kN	70 kN

Problem 11.9-23 An aluminum pipe column (alloy 6061-T6) that is fixed at the base and free at the top has outside diameter $d_2 = 3.25$ in. and inside diameter $d_1 = 3.00$ in. (see figure).

Determine the allowable axial load P_{allow} for each of the following lengths: $L = 2$ ft, 3 ft, 4 ft, and 5 ft.

Solution 11.9-23 Aluminum pipe column

Alloy 6061-T6

Fixed-free ends ($K = 2$).

$d_2 = 3.25$ in.

$d_1 = 3.00$ in.

$$A = \frac{\pi}{4}(d_2^2 - d_1^2) = 1.227 \text{ in.}^2$$

$$I = \frac{\pi}{64}(d_2^2 - d_1^2) = 1.500 \text{ in.}^4$$

$$r = \sqrt{\frac{I}{A}} = 1.106 \text{ in.}$$

Use Eqs. (11-85 *a* and *b*):

$\sigma_{allow} = 20.2 - 0.126\,(KL/r)$ ksi $\quad KL/r \leq 66$

$\sigma_{allow} = 51{,}000/(KL/r)^2$ ksi $\quad KL/r \geq 66$

L (ft)	2 ft	3 ft	4 ft	5 ft
KL/r	43.40	65.10	86.80	108.5
σ_{allow} (ksi)	14.73	12.00	6.77	4.33
$P_{allow} = \sigma_{allow} A$	18.1 k	14.7 k	8.3 k	5.3 k

Problem 11.9-24 An aluminum pipe column (alloy 6061-T6) that is fixed at the base and free at the top has outside diameter $d_2 = 80$ mm and inside diameter $d_1 = 72$ mm (see figure).

Determine the allowable axial load P_{allow} for each of the following lengths: $L = 0.6$ m, 0.8 m, 1.0 m, and 1.2 m.

(*Hint:* Convert the given data to USCS units, determine the required quantities, and then convert back to SI units.)

Solution 11.9-24 **Aluminum pipe column**

Alloy 6061-T6

Fixed-free ends ($K = 2$).

$d_2 = 80 \text{ mm} = 3.1496 \text{ in.}$

$d_1 = 72 \text{ mm} = 2.8346 \text{ in.}$

$$A = \frac{\pi}{4}(d_2^2 - d_1^2) = 1.480 \text{ in.}^2$$

$$I = \frac{\pi}{64}(d_2^2 - d_1^2) = 1.661 \text{ in.}^4$$

$$r = \sqrt{\frac{I}{A}} = 26.907 \text{ mm} = 1.059 \text{ in.}$$

Use Eqs. (11-85 *a* and *b*):

$\sigma_{allow} = 20.2 - 0.126\,(KL/r)$ ksi $\quad KL/r \le 66$

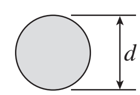
$\sigma_{allow} = 51{,}000/(KL/r)^2$ ksi $\quad KL/r \ge 66$

L (m)	0.6 m	0.8 m	1.0 m	1.2 m
KL (in.)	47.24	62.99	78.74	94.49
KL/r	44.61	59.48	74.35	89.23
σ_{allow} (ksi)	14.58	12.71	9.226	6.405
$P_{allow} = \sigma_{allow} A$	21.58 k	18.81 k	13.65 k	9.48 k
P_{allow} (kN)	96 kN	84 kN	61 kN	42 kN

Problem 11.9-25 A solid round bar of aluminum having diameter d (see figure) is compressed by an axial force $P = 60$ k. The bar has pinned supports and is made of alloy 2014-T6.

(a) If the diameter $d = 2.0$ in., what is the maximum allowable length L_{max} of the bar?

(b) If the length $L = 30$ in., what is the minimum required diameter d_{min}?



Probs. 11.9-25 through 11.9-28

Solution 11.9-25 **Aluminum bar**

Alloy 2014-T6

Pinned supports ($K = 1$). $\quad P = 60$ k

(a) FIND L_{max} IF $d = 2.0$ IN.

$$A = \frac{\pi d^2}{4} = 3.142 \text{ in.}^2 \qquad I = \frac{\pi d^4}{64}$$

$$r = \sqrt{\frac{I}{A}} = \frac{d}{4} = 0.5 \text{ in.}$$

$$\sigma_{allow} = \frac{P}{A} = \frac{60 \text{ k}}{3.142 \text{ in.}^2} = 19.10 \text{ ksi}$$

Assume L/r is less than 55:

Eq. (11-84*a*): $\sigma_{allow} = 30.7 - 0.23\,(L/r)$ ksi

or $19.10 = 30.7 - 0.23\,(L/r)$

Solve for L/r: $\frac{L}{r} = 50.43 \quad \frac{L}{r} < 55 \quad \therefore$ ok

$L_{max} = (50.43) \quad r = 25.2$ in. ←

(b) FIND d_{min} IF $L = 30$ IN.

$$A = \frac{\pi d^2}{4} \quad r = \frac{d}{4} \quad \frac{L}{r} = \frac{30 \text{ in.}}{d/4} = \frac{120 \text{ in.}}{d}$$

$$\sigma_{allow} = \frac{P}{A} = \frac{60 \text{ k}}{\pi d^2/4} = \frac{76.39}{d^2} \text{ (ksi)}$$

Assume L/r is greater than 55:

Eq. (11-84*b*): $\sigma_{allow} = \frac{54{,}000 \text{ ksi}}{(L/r)^2}$

$$\text{or} \quad \frac{76.39}{d^2} = \frac{54{,}000}{(120/d)^2}$$

$d^4 = 20.37 \text{ in.}^4 \quad d_{min} = 2.12$ in. ←

$L/r = 120/d = 120/2.12 = 56.6 > 55 \quad \therefore$ ok

Problem 11.9-26 A solid round bar of aluminum having diameter d (see figure) is compressed by an axial force $P = 175$ kN. The bar has pinned supports and is made of alloy 2014-T6.

(a) If the diameter $d = 40$ mm, what is the maximum allowable length L_{max} of the bar?

(b) If the length $L = 0.6$ m, what is the minimum required diameter d_{min}?

(*Hint:* Convert the given data to USCS units, determine the required quantities, and then convert back to SI units.)

Solution 11.9-26 Aluminum bar

Alloy 2014-T6

Pinned supports ($K = 1$). $P = 175$ kN $= 39.34$ k

(a) FIND L_{max} IF $d = 40$ MM $= 1.575$ IN.

$$A = \frac{\pi d^2}{4} = 1.948 \text{ in.}^2 \quad I = \frac{\pi d^4}{64}$$

$$r = \sqrt{\frac{I}{A}} = \frac{d}{4} = 0.3938 \text{ in.}$$

$$\sigma_{allow} = \frac{P}{A} = \frac{39.34 \text{ k}}{1.948 \text{ in.}^2} = 20.20 \text{ ksi}$$

Assume L/r is less than 55:

Eq. (11-84a): $\sigma_{allow} = 30.7 - 0.23\,(L/r)$ ksi

or $20.20 = 30.7 - 0.23\,(L/r)$

Solve for L/r: $\frac{L}{r} = 45.65 \quad \frac{L}{r} < 55 \quad \therefore$ ok

$L_{max} = (45.65)\,r = 17.98$ in. $= 457$ mm ⟵

(b) FIND d_{min} IF $L = 0.6$ m $= 23.62$ in.

$$A = \frac{\pi d^2}{4} \quad r = \frac{d}{4} \quad \frac{L}{r} = \frac{23.62 \text{ in.}}{d/4} = \frac{94.48 \text{ in.}}{d}$$

$$\sigma_{allow} = \frac{P}{A} = \frac{39.34 \text{ k}}{\pi d^2/4} = \frac{50.09}{d^2} \quad \text{(ksi)}$$

Assume L/r is greater than 55:

Eq. (11-84b): $\sigma_{allow} = \frac{54{,}000 \text{ ksi}}{(L/r)^2}$

or $\frac{50.09}{d^2} = \frac{54{,}000}{(94.48/d)^2}$

$d^4 = 8.280 \text{ in.}^4 \quad d_{min} = 1.696$ in. $= 43.1$ mm ⟵

$L/r = 94.48/d = 94.48/1.696 = 55.7 > 55 \quad \therefore$ ok

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Problem 11.9-27 A solid round bar of aluminum having diameter d (see figure) is compressed by an axial force $P = 10$ k. The bar has pinned supports and is made of alloy 6061-T6.

(a) If the diameter $d = 1.0$ in., what is the maximum allowable length L_{max} of the bar?

(b) If the length $L = 20$ in., what is the minimum required diameter d_{min}?

Solution 11.9-27 Aluminum bar

Alloy 6061-T6

Pinned Supports ($K = 1$). $P = 10$ k

(a) FIND L_{max} IF $d = 1.0$ IN.

$$A = \frac{\pi d^2}{4} = 0.7854 \text{ in.}^2 \quad I = \frac{\pi d^4}{64}$$

$$r = \sqrt{\frac{I}{A}} = \frac{d}{4} = 0.2500 \text{ in.}$$

$$\sigma_{allow} = \frac{P}{A} = \frac{10 \text{ k}}{0.7854 \text{ in.}^2} = 12.73 \text{ ksi}$$

Assume L/r is less than 66:

Eq. (11-85a): $\sigma_{allow} = 20.2 - 0.126\,(L/r)$ ksi

or $12.73 = 20.2 - 0.126\,(L/r)$

Solve For L/r: $\frac{L}{r} = 59.29 \quad \frac{L}{r} < 66 \quad \therefore$ ok

$L_{max} = (59.29)r = 14.8$ in. ⟵

(b) FIND d_{min} IF $L = 20$ in.

$$A = \frac{\pi d^2}{4} \qquad r = \frac{d}{4} \qquad \frac{L}{r} = \frac{20 \text{ in.}}{d/4} = \frac{80 \text{ in.}}{d}$$

$$\sigma_{allow} = \frac{P}{A} = \frac{10 \text{ k}}{\pi d^2/4} = \frac{12.73}{d^2} \text{ (ksi)}$$

Assume L/r is Greater than 66:

Eq. (11-85b): $\sigma_{allow} = \dfrac{51{,}000 \text{ ksi}}{(L/r)^2}$

or $\dfrac{12.73}{d^2} = \dfrac{51{,}000}{(80/d)^2}$

$d^4 = 1.597 \text{ in.}^4 \qquad d_{min} = 1.12$ in. ⟵

$L/r = 80/d = 80/1.12 = 71 > 66 \qquad \therefore$ ok

Problem 11.9-28 A solid round bar of aluminum having diameter d (see figure) is compressed by an axial force $P = 60$ kN. The bar has pinned supports and is made of alloy 6061-T6.

(a) If the diameter $d = 30$ mm, what is the maximum allowable length L_{max} of the bar?

(b) If the length $L = 0.6$ m, what is the minimum required diameter d_{min}?

(*Hint:* Convert the given data to USCS units, determine the required quantities, and then convert back to SI units.)

Solution 11.9-28 Aluminum bar

Alloy 6061-T6

Pinned Supports ($K = 1$). $\qquad P = 60 \text{ kN} = 13.49$ k

(a) FIND L_{max} IF $d = 30$ MM $= 1.181$ IN.

$$A = \frac{\pi d^2}{4} = 1.095 \text{ in.}^2 \qquad I = \frac{\pi d^4}{64}$$

$$r = \sqrt{\frac{I}{A}} = \frac{d}{4} = 0.2953 \text{ in.}$$

$$\sigma_{allow} = \frac{P}{A} = \frac{13.49 \text{ k}}{1.095 \text{ in.}^2} = 12.32 \text{ ksi}$$

Assume L/r is less than 66:

Eq. (11-85a): $\sigma_{allow} = 20.2 - 0.126\,(L/r)$ ksi

or $12.32 = 20.2 - 0.126\,(L/r)$

Solve For L/r: $\dfrac{L}{r} = 62.54 \qquad \dfrac{L}{r} < 66 \qquad \therefore$ ok

$L_{max} = (62.54)r = 18.47 \text{ in.} = 469$ mm ⟵

(b) FIND d_{min} IF $L = 0.6$ M $= 23.62$ IN.

$$A = \frac{\pi d^2}{4} \qquad r = \frac{d}{4} \qquad \frac{L}{r} = \frac{23.62 \text{ in.}}{d/4} = \frac{94.48 \text{ in.}}{d}$$

$$\sigma_{allow} = \frac{P}{A} = \frac{13.48 \text{ k}}{\pi d^2/4} = \frac{17.18}{d^2} \text{ (ksi)}$$

Assume L/r is Greater than 66:

Eq. (11-85b): $\sigma_{allow} = \dfrac{51{,}000 \text{ ksi}}{(L/r)^2}$

or $\dfrac{17.18}{d^2} = \dfrac{51{,}000}{(94.48/d)^2}$

$d^4 = 3.007 \text{ in.}^4 \quad d_{min} = 1.317 \text{ in.} = 33.4$ mm ⟵

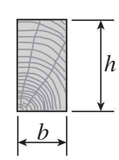
$L/r = 94.48/d = 94.48/1.317 = 72 > 66 \qquad \therefore$ ok

Wood Columns

When solving the problems for wood columns, assume that the columns are constructed of sawn lumber (c = 0.8 and K_{cE} = 0.3) and have pinned-end conditions. Also, buckling may occur about either principal axis of the cross section.

Problem 11.9-29 A wood post of rectangular cross section (see figure) is constructed of 4 in. × 6 in. structural grade, Douglas fir lumber ($F_c = 2{,}000$ psi, $E = 1{,}800{,}00$ psi). The net cross-sectional dimensions of the post are $b = 3.5$ in. and $h = 5.5$ in. (see Appendix F).

Determine the allowable axial load P_{allow} for each of the following lengths: $L = 5.0$ ft, 7.5 ft, and 10.0 ft.



Probs. 11.9-29 through 11.9-32

Solution 11.9-29 **Wood post (rectangular cross section)**

$F_c = 2{,}000$ psi $E = 1{,}800{,}000$ psi $c = 0.8$

$K_{cE} = 0.3$ $b = 3.5$ in. $h = 5.5$ in. $d = b$

Find P_{allow}

Eq. (11-94): $\phi = \dfrac{K_{cE}E}{F_c(L_e/d)^2}$

Eq. (11-95): $C_P = \dfrac{1+\phi}{2c} - \sqrt{\left[\dfrac{1+\phi}{2c}\right]^2 - \dfrac{\phi}{c}}$

Eq. (11-92): $P_{allow} = F_cC_PA = F_cC_Pbh$

L_e	5 ft	7.5 ft	10.0 ft
L_e/d	17.14	25.71	34.29
ϕ	0.9188	0.4083	0.2297
C_P	0.6610	0.3661	0.2176
P_{allow}	25.4 k	14.1 k	8.4 k

Problem 11.9-30 A wood post of rectangular cross section (see figure) is constructed of structural grade, southern pine lumber ($F_c = 14$ MPa, $E = 12$ GPa). The cross-sectional dimensions of the post (actual dimensions) are $b = 100$ mm and $h = 150$ mm.

Determine the allowable axial load P_{allow} for each of the following lengths: $L = 1.5$ m, 2.0 m, and 2.5 m.

Solution 11.9-30 **Wood post (rectangular cross section)**

$F_c = 14$ MPa $E = 12$ GPa $c = 0.8$ $K_{cE} = 0.3$

$b = 100$ mm $h = 150$ mm $d = b$

Find P_{allow}

Eq. (11-94): $\phi = \dfrac{K_{cE}E}{F_c(L_e/d)^2}$

Eq. (11-95): $C_P = \dfrac{1+\phi}{2c} - \sqrt{\left[\dfrac{1+\phi}{2c}\right]^2 - \dfrac{\phi}{c}}$

Eq. (11-92): $P_{allow} = F_cC_PA = F_cC_Pbh$

L_e	1.5 m	2.0 m	2.5 m
L_e/d	15	20	25
ϕ	1.1429	0.6429	0.4114
C_P	0.7350	0.5261	0.3684
P_{allow}	154 kN	110 kN	77 kN ←

Problem 11.9-31 A wood column of rectangular cross section (see figure) is constructed of 4 in. × 8 in. construction grade, western hemlock lumber ($F_c = 1{,}000$ psi, $E = 1{,}300{,}000$ psi). The net cross-sectional dimensions of the column are $b = 3.5$ in. and $h = 7.25$ in. (see Appendix F).

Determine the allowable axial load P_{allow} for each of the following lengths: $L = 6$ ft, 8 ft, and 10 ft.

Solution 11.9-31 **Wood column (rectangular cross section)**

$F_c = 1{,}000$ psi $E = 1{,}300{,}000$ psi $c = 0.8$

$K_{cE} = 0.3$ $b = 3.5$ in. $h = 7.25$ in. $d = b$

Find P_{allow}

Eq. (11-94): $\phi = \dfrac{K_{cE}E}{F_c(L_e/d)^2}$

Eq. (11-95): $C_P = \dfrac{1+\phi}{2c} - \sqrt{\left[\dfrac{1+\phi}{2c}\right]^2 - \dfrac{\phi}{c}}$

Eq. (11-92): $P_{allow} = F_cC_FA = F_cC_Pbh$

L_e	6 ft	8 ft	10 ft
L_e/d	20.57	27.43	34.29
ϕ	0.9216	0.5184	0.3318
C_P	0.6621	0.4464	0.3050
P_{allow}	16.8 k	11.3 k	7.7 k ←

Problem 11.9-32 A wood column of rectangular cross section (see figure) is constructed of structural grade, Douglas fir lumber ($F_c = 12$ MPa, $E = 10$ GPa). The cross-sectional dimensions of the column (actual dimensions) are $b = 140$ mm and $h = 210$ mm.

Determine the allowable axial load P_{allow} for each of the following lengths: $L = 2.5$ m, 3.5 m, and 4.5 m.

Solution 11.9-32 **Wood column (rectangular cross section)**

$F_c = 12$ MPa $\quad E = 10$ GPa $\quad c = 0.8 \quad K_{cE} = 0.3$

$b = 140$ mm $\quad h = 210$ mm $\quad d = b$

Find P_{allow}

Eq. (11-94): $$\phi = \frac{K_{cE}E}{F_c(L_e/d)^2}$$

Eq. (11-95): $$C_P = \frac{1+\phi}{2c} - \sqrt{\left[\frac{1+\phi}{2c}\right]^2 - \frac{\phi}{c}}$$

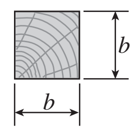
Eq. (11-92): $$P_{allow} = F_c C_P A = F_c C_P bh$$

L_e	2.5 m	3.5 m	4.5 m
L_e/d	17.86	25.00	32.14
ϕ	0.7840	0.4000	0.2420
C_P	0.6019	0.3596	0.2284
P_{allow}	212 kN	127 kN	81 kN ⟵

Problem 11.9-33 A square wood column with side dimensions b (see figure) is constructed of a structural grade of Douglas fir for which $F_c = 1{,}700$ psi and $E = 1{,}400{,}000$ psi. An axial force $P = 40$ k acts on the column.

(a) If the dimension $b = 5.5$ in., what is the maximum allowable length L_{max} of the column?

(b) If the length $L = 11$ ft, what is the minimum required dimension b_{min}?



Probs. 11.9-33 through 11.9-36

Solution 11.9-33 **Wood column (square cross section)**

$F_c = 1{,}700$ psi $\quad E = 1{,}400{,}000$ psi $\quad c = 0.8$

$K_{cE} = 0.3 \quad P = 40$ k

(a) Maximum length L_{max} for $b = d = 5.5$ in.

From Eq. (11-92): $$C_P = \frac{P}{F_c b^2} = 0.77783$$

From Eq. (11-95):

$$C_P = 0.77783 = \frac{1+\phi}{1.6} - \sqrt{\left[\frac{1+\phi}{1.6}\right]^2 - \frac{\phi}{0.8}}$$

Trial and error: $\phi = 1.3225$

From Eq. (11-94): $$\frac{L}{d} = \sqrt{\frac{K_{cE}E}{\phi F_c}} = 13.67$$

$\therefore L_{max} = 13.67\ d = (13.67)(5.5 \text{ in.})$

$= 75.2$ in. ⟵

(b) Minimum dimension b_{min} for $L = 11$ ft

Trial and error: $$\frac{L}{d} = \frac{L}{b} \quad \phi = \frac{K_{cE}E}{F_c(L/d)^2}$$

$$C_P = \frac{1+\phi}{1.6} - \sqrt{\left[\frac{1+\phi}{1.6}\right]^2 - \frac{\phi}{0.8}} \quad P = F_c C_P b^2$$

Given load: $P = 40$ k

Trial b (in.)	$\frac{L}{d} = \frac{L}{b}$	ϕ	C_P	P (kips)
6.50	20.308	0.59907	0.49942	35.87
6.70	19.701	0.63651	0.52230	39.86
6.71	19.672	0.63841	0.52343	40.06

$\therefore b_{min} = 6.71$ in. ⟵

Problem 11.9-34 A square wood column with side dimensions b (see figure) is constructed of a structural grade of southern pine for which $F_c = 10.5$ MPa and $E = 12$ GPa. An axial force $P = 200$ kN acts on the column.

(a) If the dimension $b = 150$ mm, what is the maximum allowable length L_{max} of the column?

(b) If the length $L = 4.0$ m, what is the minimum required dimension b_{min}?

Solution 11.9-34 **Wood column (square cross section)**

$F_c = 10.5$ MPa $\quad E = 12$ GPa $\quad c = 0.8$

$K_{cE} = 0.3 \quad P = 200$ kN

(a) Maximum length L_{max} for $b = d = 150$ mm

From Eq. (11-92): $C_P = \dfrac{P}{F_c b^2} = 0.84656$

From Eq. (11-95):

$$C_P = 0.84656 = \frac{1+\phi}{1.6} - \sqrt{\left[\frac{1+\phi}{1.6}\right]^2 - \frac{\phi}{0.8}}$$

Trial and error: $\phi = 1.7807$

From Eq. (11-94): $\dfrac{L}{d} = \sqrt{\dfrac{K_{cE}E}{\phi F_c}} = 13.876$

$\therefore L_{max} = 13.876\, d = (13.876)(150 \text{ mm})$
$= 2.08$ m ⟵

(b) Minimum dimension b_{min} for $L = 4.0$ m

Trial and error: $\dfrac{L}{d} = \dfrac{L}{b} \quad \phi = \dfrac{K_{cE}E}{F_c(L/d)^2}$

$$C_P = \frac{1+\phi}{1.6} - \sqrt{\left[\frac{1+\phi}{1.6}\right]^2 - \frac{\phi}{0.8}} \qquad P = F_c C_P b^2$$

Given load: $P = 200$ kN

Trial b (mm)	$\frac{L}{d} = \frac{L}{b}$	ϕ	C_P	P (kN)
180	22.22	0.69429	0.55547	189.0
182	21.98	0.70980	0.56394	196.1
183	21.86	0.71762	0.56814	199.8
184	21.74	0.72549	0.57231	203.5

$\therefore b_{min} = 184$ mm ⟵

Problem 11.9-35 A square wood column with side dimensions b (see figure) is constructed of a structural grade of spruce for which $F_c = 900$ psi and $E = 1{,}500{,}000$ psi. An axial force $P = 8.0$ k acts on the column.

(a) If the dimension $b = 3.5$ in., what is the maximum allowable length L_{max} of the column?

(b) If the length $L = 10$ ft, what is the minimum required dimension b_{min}?

Solution 11.9-35 **Wood column (square cross section)**

$F_c = 900$ psi $\quad E = 1{,}500{,}000$ psi $\quad c = 0.8$

$K_{cE} = 0.3 \quad P = 8.0$ k

(a) Maximum length L_{max} for $b = d = 3.5$ in.

From Eq. (11-92): $C_P = \dfrac{P}{F_c b^2} = 0.72562$

From Eq. (11-95):

$$C_P = 0.72562 = \frac{1+\phi}{1.6} - \sqrt{\left[\frac{1+\phi}{1.6}\right]^2 - \frac{\phi}{0.8}}$$

Trial and error: $\phi = 1.1094$

From Eq. (11-94): $\dfrac{L}{d} = \sqrt{\dfrac{K_{cE}E}{\phi F_c}} = 21.23$

$\therefore L_{max} = 21.23\, d = (21.23)(3.5 \text{ in.}) = 74.3$ in. ⟵

(b) MINIMUM DIMENSION b_{min} FOR $L = 10$ FT

Trial and error. $\frac{L}{d} = \frac{L}{b}$ $\phi = \frac{K_{cE}E}{F_c(L/d)^2}$

$$C_P = \frac{1+\phi}{1.6} - \sqrt{\left[\frac{1+\phi}{1.6}\right]^2 - \frac{\phi}{0.8}} \qquad P = F_c C_P b^2$$

Given load: $P = 8000$ lb

$\therefore b_{min} = 4.20$ in. ⟵

Trial b (in.)	$\frac{L}{d} = \frac{L}{b}$	ϕ	C_P	P (lb)
4.00	30.00	0.55556	0.47145	6789
4.20	28.57	0.61250	0.50775	8061
4.19	28.64	0.60959	0.50596	7994

Problem 11.9-36 A square wood column with side dimensions b (see figure) is constructed of a structural grade of eastern white pine for which $F_c = 8.0$ MPa and $E = 8.5$ GPa. An axial force $P = 100$ kN acts on the column.

(a) If the dimension $b = 120$ mm, what is the maximum allowable length L_{max} of the column?

(b) If the length $L = 4.0$ m, what is the minimum required dimension b_{min}?

Solution 11.9-36 **Wood column (square cross section)**

$F_c = 8.0$ MPa $\quad E = 8.5$ GPa $\quad c = 0.8$

$K_{cE} = 0.3 \quad P = 100$ kN

(a) MAXIMUM LENGTH L_{max} FOR $b = d = 120$ mm

From Eq. (11-92): $C_P = \frac{P}{F_c b^2} = 0.86806$

From Eq. (11-95):

$$C_P = 0.86806 = \frac{1+\phi}{1.6} - \sqrt{\left[\frac{1+\phi}{1.6}\right]^2 - \frac{\phi}{0.8}}$$

Trial and error: $\phi = 2.0102$

From Eq. (11-94): $\frac{L}{d} = \sqrt{\frac{K_{cE}E}{\phi F_c}} = 12.592$

$\therefore L_{max} = 12.592\, d = (12.592)(120 \text{ mm})$

$= 1.51$ m ⟵

(b) MINIMUM DIMENSION b_{min} FOR $L = 4.0$ m

Trial and error. $\frac{L}{d} = \frac{L}{b}$ $\phi = \frac{K_{cE}E}{F_c(L/d)^2}$

$$C_P = \frac{1+\phi}{1.6} - \sqrt{\left[\frac{1+\phi}{1.6}\right]^2 - \frac{\phi}{0.8}} \qquad P = F_c C_P b^2$$

Given load: $P = 100$ kN

Trial b (mm)	$\frac{L}{d} = \frac{L}{b}$	ϕ	C_P	P (kN)
160	25.00	0.51000	0.44060	90.23
164	24.39	0.53582	0.45828	98.61
165	24.24	0.54237	0.46269	100.77

$\therefore b_{min} = 165$ mm ⟵